FYS3610

EXERCISES WEEK 38

Midterm exam from autumn 2006. You should be able to answer all questions except problem 2e.

DEPARTMENT OF PHYSICS, UIO

FYS3610-SPACE PHYSICS

MID-TERM EXAMINATION

Date: October 9, 2006

Time of day: 13:30-15:30 (2 hours!)

Permitted aid(s): Calculating machine.

The set consists of 4 pages, with 3 Problems.

NOTE: At page 4 you find a Table containing useful information.

PROBLEM 1

a) Sketch a typical electron density (m^{-3}) versus altitude profile for daytime sunlit conditions. Mark out the altitude ranges for the D-, E-, and F-layers.

The ion and electron momentum equations are given as:

$$n_i m_i \frac{d\vec{v}_i}{dt} = n_i e(\vec{E} + \vec{v}_i \times \vec{B}) - n_i m_i \nu_{in} \vec{v}_i \qquad (1.1)$$

$$n_e m_e \frac{d\vec{v}_e}{dt} = -n_e e(\vec{E} + \vec{v}_e \times \vec{B}) - n_e m_e \nu_{en} \vec{v}_e \qquad (1.2)$$

b) Describe the different terms in Eq. 1.1. Explain briefly how the following expressions for $\vec{v}_e$ and $\vec{v}_i$ can be derived:

$$\vec{v}_i = \frac{\omega_i \nu_{in}}{\omega_i^2 + \nu_{in}^2} \frac{\vec{E}_\perp}{B} + \frac{\omega_i^2}{\omega_i^2 + \nu_{in}^2} \frac{\vec{E}_\perp \times \vec{B}}{B^2} \qquad (1.3)$$

$$\vec{v}_e = -\frac{\omega_e \nu_{en}}{\omega_e^2 + \nu_{en}^2} \frac{\vec{E}_\perp}{B} + \frac{\omega_e^2}{\omega_e^2 + \nu_{en}^2} \frac{\vec{E}_\perp \times \vec{B}}{B^2} \qquad (1.4)$$

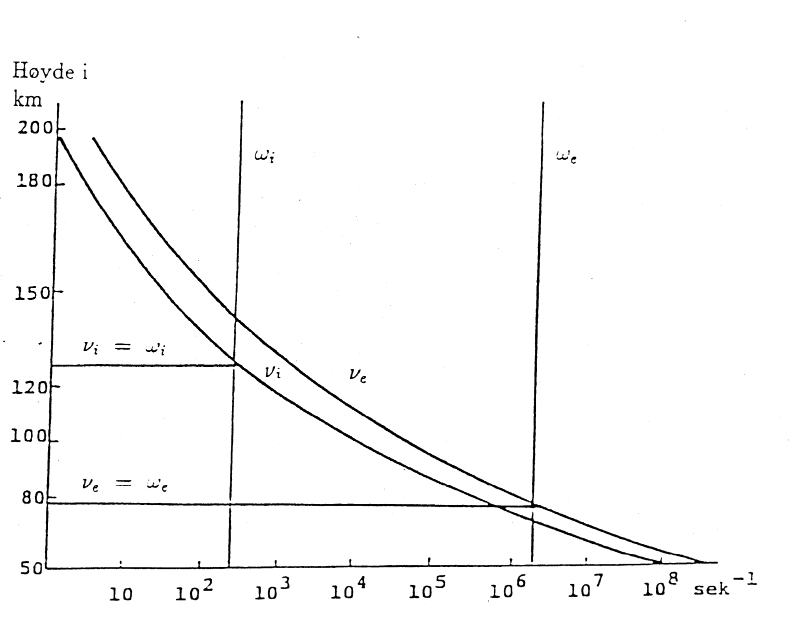

Figure 1.1

c) Explain Figure 1.1 above.

d) Apply Figure 1.1, Eq. 1.3 and Eq. 1.4 to describe rotation of the velocity vectors $\vec{v}_i$ and $\vec{v}_e$ by altitude. (Hint: Derive expressions for the angle between the velocity vectors and $\vec{E}_\perp$ and draw an illustration). Derive expressions for the electron and the ion speeds and comment on the altitude variation. What is the maximal speed?

e) Find an expression for the current density $\vec{j}$. Point out the Hall and Pedersen terms and discuss j versus altitude. What controls the upper and the lower limits of the conductive layer?

PROBLEM 2

$$\nabla \times \vec{B} = \mu_0 \vec{j} \qquad (2.1)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial\, t} \qquad (2.2)$$

$$\vec{j} = \sigma(\vec{E} + \vec{v} \times \vec{B}) \qquad (2.3)$$

a) What are the well-known names of Eqs. 2.1-2.3. Define the parameters involved.

b) Derive and expression for $\frac{\partial \vec{B}}{\partial\, t}$ and show that the magnetic Reynold's number is given by $R_m = \mu_0 \sigma\ vL$.

c) Discuss the physical implications of the $R_m << 1$ and $R_m >> 1$.

d) Make a brief discussion of the frozen-in-field concept. Give an example where the frozen-in-field concept breaks down.

e) Give a cartoon description of a CME event.

PROBLEM 3

Suppose that the earth's magnetic field is 3×10^{-5} T at the equator and falls off as $1/r^{-3}$ as for a perfect dipole. Let there be an isotropic population of 1-eV protons and 30-keV electrons, each with a density of $n = 10^7\ m^{-3}$ at r = 5 earth radii in the equatorial plane. The general expression for the gradient drift is given as:

$$\vec{u}_{\nabla B} = \frac{1}{2} m v_\perp^2 \frac{\vec{B} \times \nabla \vec{B}}{qB^3} \qquad \text{Eq. 3.1}$$

a) Compute the ion and electron gradient drift velocities.

b) Does the electron drift eastward or westward?

c) How long does it take for the electron to encircle the earth?

d) Compute the ring current density Am^{-2}.

Table:

$$E = \frac{1}{2} m v_{\perp}^{2}$$

$$1\,\mathrm{eV} = 1.6 \times 10^{-19}\ \mathrm{J}$$

$$\mathrm{B}_{\mathrm{Eq}} = \mathrm{B}_0 \left(\frac{\mathrm{R}_{\mathrm{E}}}{\mathrm{r}}\right)^3$$

$$\mathrm{B}_0 = 30000\ \mathrm{nT}$$

$$\mathrm{R}_{\mathrm{E}} = 6400\,\mathrm{km}$$

$$\mathrm{m}_{\mathrm{p}} = 1.67 \times 10^{-27}\,\mathrm{kg}$$

$$\mathrm{m}_{\mathrm{e}} = 9.1 \times 10^{-31}\ \mathrm{kg}$$

$$\mathrm{c} = 3 \times 10^{8}\,\mathrm{ms}^{-1}$$

$$\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$$

$$\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) + \nabla^2 \vec{A}$$